BRIEF COMMUNICATIONS

HEAT TRANSFER ON A PLATE IN A TURBULENT FLOW WITH CONSTANT SURFACE HEAT FLUX AND AN ISOTHERMAL WALL

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Numerous experimental investigations of heat transfer are conducted under conditions either of constant surface heat flux or of constant wall temperature. Thus, the question arises: in what circumstances can one compare data obtained under different experimental conditions? In one of the papers [5] devoted to this question, it was shown experimentally that heat transfer in the turbulent flow regime in tubes with constant wall temperature and with constant heat flux is comparable, i.e., identical.

The article mentioned makes a comparative appraisal, by theoretical and experimental means, of heat transfer on a plate with constant wall temperature and with constant surface heat flux in the turbulent flow regime.

Experimental investigations of local heat transfer on a plate in a turbulent stream of air were carried out in calorimeter plates with an isothermal wall or with constant surface heat flux.

The results of reducing the heat transfer test data in all the cases are presented by a correlation of the form

$$\mathrm{Nu}_x = A\,\mathrm{Re}_x^{0.8}. \tag{1}$$

In [1] an experimental determination was made of local heat transfer on a plate with constant surface heat flux, and a value of A equal to 0.0255 was obtained. In [2] reduction of experimental data on local heat transfer on a plate with constant wall temperature gave A = 0.0235. The reduced value of A is attributed in [2] to more careful calculation of heat losses in the experimental investigation.

The equation of the thermal boundary layer for a plate with constant wall temperature has the form

$$\frac{d\delta_T^{**}}{dx} + \frac{\Theta'}{\Theta}\delta_T^{**} = \frac{q_W}{c_p\gamma w_\infty\Theta}. \tag{2}$$

An approximate solution of (2) was obtained by the method described by Ambrok [4], on the assumption that the heat transfer law does not depend on the pressure gradient or on the temperature distribution along the surface. By the heat transfer law we mean a functional relation of the type

$$q_W/c_p\gamma w_\infty\Theta = f(\mathrm{Re}_T^{**}), \tag{3}$$

where $\mathrm{Re}_T^{**} = W_\infty\delta_T^{**}/\nu$.

The heat transfer law (3) for a plate with constant surface temperature may be obtained by analysis of equation (2). For this case, we transform (2), when $\Theta' = 0$, to the form

$$d\delta_T^{**}/dx = q_W/c_p\gamma w_\infty\Theta. \tag{4}$$

The left side of (4) is

$$q_W/c_p\gamma w_\infty\Theta = \mathrm{Nu}_x/\mathrm{Pr}\,\mathrm{Re}_x. \tag{5}$$

If we use an experimental or theoretical relation

$$\mathrm{Nu}_x = A\,\mathrm{Re}_x^n \tag{6}$$

for the plate with isothermal wall, then (5) takes the form

$$\frac{q_W}{c_p\gamma w_\infty\Theta} = \frac{A}{\mathrm{Pr}}\mathrm{Re}_x^{n-1}. \tag{7}$$

Making use of the last formula, and integrating the differential equation (4) in the range 0 to x, we obtain an expression for δ_T^{**},

$$\delta_T^{**} = \frac{A}{\mathrm{Pr}\,n}\frac{\nu}{w_\infty}\mathrm{Re}_x^n, \tag{8}$$

in which

$$\mathrm{Re}_x = \left(\frac{\mathrm{Pr}\,n}{A}\right)^{\frac{1}{n}}\mathrm{Re}_T^{**\frac{1}{n}}. \tag{9}$$

Substituting Re_x into (7), we obtain the heat transfer law for a plate with constant surface temperature:

$$\frac{q_W}{c_p\gamma w_\infty\Theta} = \left(\frac{n^{n-1}A}{\mathrm{Pr}}\right)^{\frac{1}{n}}\mathrm{Re}_T^{**\frac{n-1}{n}}. \tag{10}$$

The law obtained is used in solving the differential equation (2) for determining the heat transfer on a plate with variable wall temperature. For this we substitute (10) into (2), to obtain an equation for determining δ_T^{**}:

$$\frac{d\delta_T^{**}}{dx} + \frac{\Theta'}{\Theta}\delta_T^{**} = \left(\frac{n^{n-1}A}{\mathrm{Pr}}\right)\mathrm{Re}_T^{**\frac{n-1}{n}}. \tag{11}$$

The solution of the last equation has the form

$$\delta_T^{**} = \left[\left(\frac{A}{n\mathrm{Pr}}\right)^{\frac{1}{n}}\frac{w_\infty^{\frac{n-1}{n}}}{\Theta^{\frac{1}{n}}\nu^{\frac{n-1}{n}}}\int_{x_0}^{x}\Theta^{\frac{1}{n}}\,dx + \frac{C_1}{\Theta^{\frac{1}{n}}}\right]^n. \tag{12}$$

When $x = x_0 = 0$, $C_1 = 0$, Eq. (12) is written as

$$\delta_T^{**}\Theta = \frac{A}{n\,\mathrm{Pr}}\left(\frac{w_\infty}{\nu}\right)^{n-1}\left(\int_0^x\Theta^{\frac{1}{n}}\,dx\right)^n. \tag{13}$$

Thus, we obtain the law of variation of δ_T^{**} along a plate with variable temperature. Carrying out the substitution of δ_T^{**} from (13) into (2), we obtain

$$\frac{q_W}{c_p\gamma w_\infty} = \frac{A}{\mathrm{Pr}}\left(\frac{w_\infty}{\nu}\right)^{n-1}\left(\int_0^x\Theta^{\frac{1}{n}}\,dx\right)^{n-1}\Theta^{\frac{1}{n}}, \tag{14}$$

or in parametric form

$$\mathrm{Nu}_x = A\,\mathrm{Re}_x^n K_{\mathrm{V.T}}, \tag{15}$$

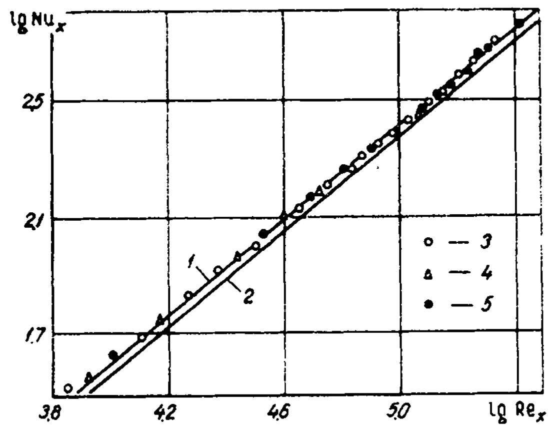
where

$$K_{\mathrm{V.T}} = \left(\Theta^{\frac{1}{n}}x\Big/\int_0^x\Theta^{\frac{1}{n}}\,dx\right)^{1-n}. \tag{16}$$

Equations (15) and (16) are suitable for calculation of the local heat transfer on a plate with variable wall temperature for any boundary layer structure. Here the condition must be observed that the start of the boundary layer of identical structure coincides with the beginning of the heated section of the plate.

We will make a full examination of the turbulent boundary layer on the plate. If we use the experimental relation $\mathrm{Nu}_x = 0.0235\,\mathrm{Re}_x^{0.8}$ for heat transfer on a plate with t = const, then (15) transforms to the calculation formula

$$\mathrm{Nu}_x = 0.0235\,\mathrm{Re}_x^{0.8}K_{\mathrm{V.T}}. \tag{17}$$



Heat transfer on a plate in the turbulent flow regime according to the experimental data of [1] (1) and of [2] (2), and calculated from (17) using the measured wall temperature for oncoming stream velocities of w_∞ = 6.8 m/sec (3), 9 (4) and 14.2 (5).

where

$$K_{V.T} = \left(\Theta^{1.25}x/\int_0^x \Theta^{1.25}\, dx\right)^{0.2}. \qquad (18)$$

If the curve $\Theta^{1.25} = f(x)$ is convex, which occurs for heat transfer on a plate with q = const on its surface, then the inequality

$$\Theta^{1.25}\, x/\int_0^x \Theta^{1.25}\, dx \leqslant 2 \qquad (19)$$

becomes evident. Then the quantity $K_{V.T}$ will lie in the range $1 < K_{V.T} < \sqrt[5]{2} = 1.15$.

Thus it may be seen that the local value of Nusselt number for a plate with q = const is greater than the local values of Nu_x for a plate with t = const by no more than 15% at the initial section of the heater, the difference in heat transfer falling off with distance from the beginning of the heater. This is confirmed by the results of [1, 2], in which it is shown that with t = const the heat transfer on the plate is 8% less than with q = const, i.e., A = 0.0255.

It follows from (15) and (16) that in a laminar flow, heat transfer on a plate with constant heat flux and heat transfer at constant wall temperature differ appreciably. Since n = 0.5 for laminar flow, this difference may reach 40%.

An experimental verification of (17) was done on an experimental rig consisting of a working section connected into the suction tube of a blowdown wind tunnel. This equipment permits measurement of local surface temperature of the plate (28 copper-constantan thermocouples) and measurement of velocity distribution in the boundary layer (by pressure measurement tubes and an electrical anemometer).

The calorimeter plate made of micarta was 500 × 198 × 20. The calorimeter was heated by means of 72 identical nichrome heaters (36 each on the upper and lower surfaces of the plate) to achieve a uniform heat flux at the plate surface.

In order to ensure a completely turbulent boundary layer, as in [2], we used a turbulence generator in the form of a thin wire upstream of the heated section of the plate.

The figure shows the calculated local heat transfer on a plate according to (17). In accordance with (18), the coefficient $k_{V.T}$ was determined from the measured temperature distribution along the plate surface.

In conclusion it may be added that the theoretical solution obtained in [3] for heat transfer on a plate with an isothermal wall is in good agreement with the test data of [2]. Thus, the relation $Nu_x = 0.0235 \cdot Re_x^{0.8}$ [2] used for the plate with t = const may be considered to be quite well founded, both theoretically and experimentally.

NOTATION

$Nu_x = \alpha x/\lambda$, $Re_x = w_\infty x/\nu$ are the local values of Nusselt and Reynolds numbers; x is the distance from the beginning of the heated plate to the variable section at which the heat transfer coefficient takes the value α; w_∞ is the velocity of oncoming stream; A is the experimental constant; $\Theta = t_w - t_0$ is the variable temperature head, the difference between the wall temperature t_w and the stream temperature t_0; q_w is the heat flux at plate surface washed by stream; $\delta_T^{**} = \int_0^{\delta_T} \frac{w_x}{w_\infty} \cdot \left(1 - \frac{t_w - t}{t_w - t_0}\right) dy$ is the enthalpy thickness; $\Theta' = d\Theta/dx$; δ_T is the thermal boundary layer thickness; t is the temperature in thermal boundary layer; w_x is the velocity in boundary layer; $Pr = \gamma c_p \nu/\lambda$ is the Prandtl number; $\alpha = q_w/\Theta$.

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